

Cycling Situations

A-E Strand(s): Algebra. Sample Courses: Middle School Course 2, Middle School One-Year Advanced Course, Integrated 1, and Algebra I.

Topic/Expectation

A.A.6 Linear equations and inequalities

- d. Use graphs to help solve linear equations in one variable.
- g. Recognize and solve problems that can be modeled using linear equations or inequalities in one or two variables; interpret the solution(s) in terms of the context of the problem.

A.B.3 Systems of linear equations and inequalities

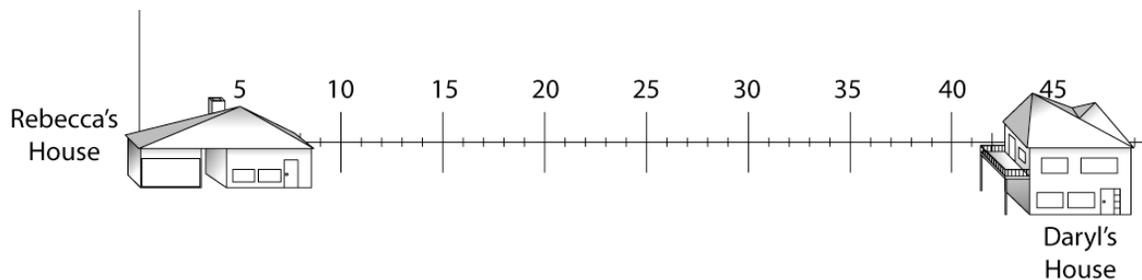
- a. Solve systems of simultaneous linear equations in two variables using algebraic procedures.
- b. Use graphs to help solve systems of simultaneous linear equations in two variables.

Rationale

This task, accessible to beginning algebra students, builds an understanding of what it means to solve systems of linear equations by using diagrams, tables, and graphs. It introduces students to systems of equations in a way that may sharpen their understanding and encourage them to explore multiple representations to solve problems. In later instruction, students will extend their knowledge of solving linear systems to include algebraic methods and to solve systems of non-linear equations.

Instructional Task

1. Rebecca's and Daryl's houses are located on a 50-mile bicycle course. Rebecca's house is located at the 5-mile marker, and Daryl's house is at the 45-mile marker. Rebecca leaves her house and begins riding towards Daryl's house at a constant rate of 11 miles per hour. At the same time, Daryl leaves his house and begins riding toward Rebecca's house at a constant rate of 9 miles per hour. Mark the course on the picture provided below to show the distances that Rebecca and Daryl have traveled after each hour, and then complete the table of values for each rider. Use the process column to describe what is happening at each hour. This may help you create an algebraic rule.



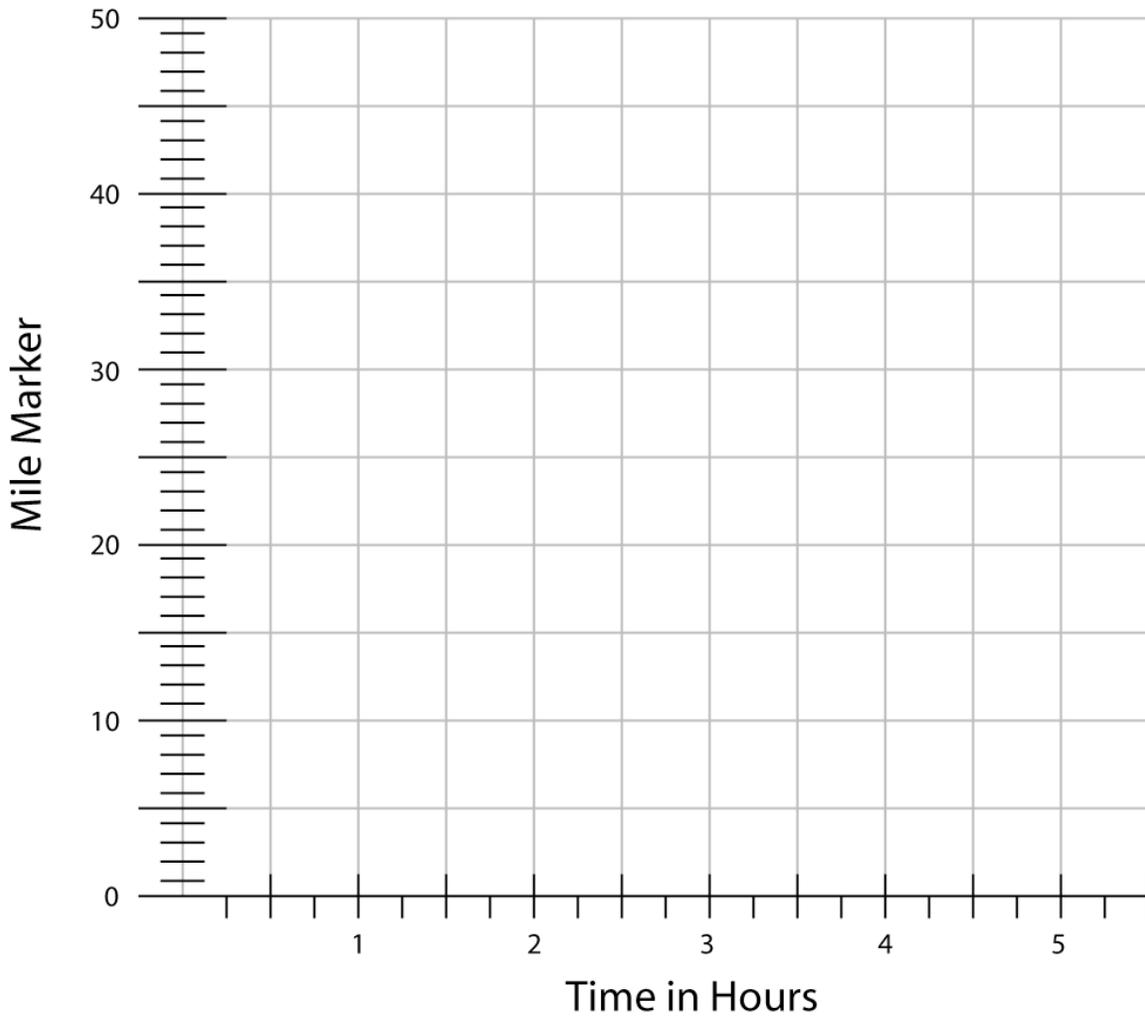
Rebecca's Trip

Hour	Process	Mile Marker
0		
1		
2		
3		
4		
5		
h		

Daryl's Trip

Hour	Process	Mile Marker
0		
1		
2		
3		
4		
5		
h		

2. What are the independent and dependent quantities in this situation? Explain your thinking.
3. How long will it take Rebecca to get to Daryl's house?
4. How long will it take Daryl to get to Rebecca's house?
5. Write a sentence and a function rule describing Rebecca's distance from mile marker 0 in relation to elapsed time.
6. Write a sentence and a function rule describing Daryl's distance from mile marker 0 in relation to elapsed time.
7. On the graph below, plot the points from both tables and label each point with an ordered pair.
8. When are Rebecca and Daryl at the same place on the course? How can you tell from the graph?
9. Write an equation to determine how long it takes for Rebecca and Daryl to be at the same place on the course. What does this equation represent on the graph?



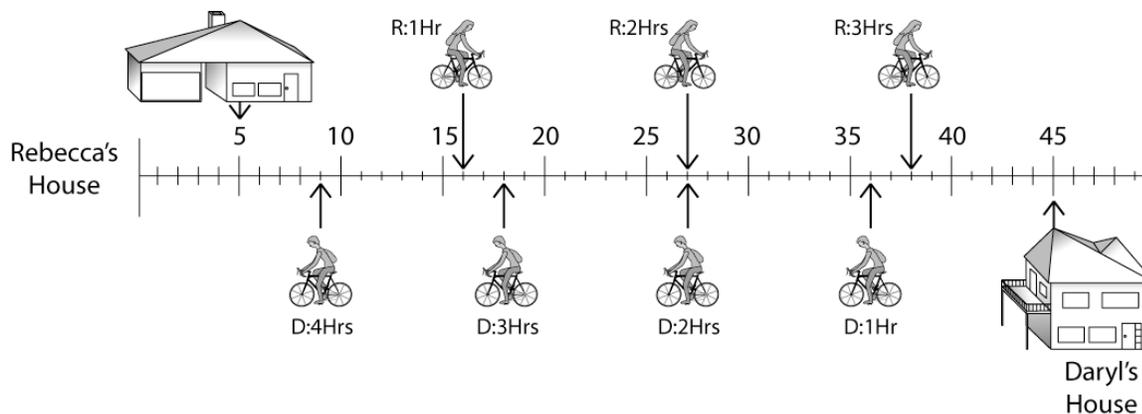
Discussion/Further Questions/Extensions

The Process column in the table allows students to record their thinking and/or the mathematical process they used to arrive at the value for the mile marker. This provides a good foundation for finding a function rule.

Sample Solutions

1. Rebecca’s and Daryl’s houses are located on a 50-mile bicycle course. Rebecca’s house is located at the 5-mile marker, and Daryl’s house is at the 45-mile marker. Rebecca leaves her house and begins riding toward Daryl’s house at a constant rate of 11 miles per hour. At the same time, Daryl leaves his house and begins riding toward Rebecca’s house at a constant rate of 9 miles per hour. Mark the course on the picture provided below to show the distances that Rebecca and Daryl have traveled after each hour, and then complete the table of values for each rider. Use the process column to describe what is happening at each hour. This may help you create an algebraic rule.

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Rebecca's Trip

Hour	Process	Mile Marker
0	$5 + 11(0)$	5
1	$5 + 11(1)$	16
2	$5 + 11(2)$	27
3	$5 + 11(3)$	38
4	$5 + 11(4)$	49
5	$5 + 11(5)$	60
h	$5 + 11(h)$	$5 + 11h$

Daryl's Trip

Hour	Process	Mile Marker
0	$45 - 9(0)$	45
1	$45 - 9(1)$	36
2	$45 - 9(2)$	27
3	$45 - 9(3)$	18
4	$45 - 9(4)$	9
5	$45 - 9(5)$	0
H	$45 - 9(h)$	$45 - 9h$

2. What are the independent and dependent quantities in this situation? Explain your thinking.

In this situation, it makes sense that the independent quantity is the number of hours spent bike riding, and the dependent quantity is the mile marker where the biker is located since you are determining the mile marker when given the number of hours. Note that while these answers are the most likely for this situation, they could conceivably be reversed if the student could provide an appropriate justification.

3. How long will it take Rebecca to get to Daryl's house?

Looking at the drawing, we can estimate the time to be between 3 and 4 hours. To determine exactly how long it takes, students could solve an equation or graph on a graphing calculator by entering the function rules.

a) Solve $5 + 11h = 45$; $h = 3.64$

b) $Y_1 = 5 + 11h$ and $Y_2 = 45$

Find the intersection of the two functions to determine that it takes about 3.64 hours for Rebecca to get to Daryl's house.

4. How long will it take Daryl to get to Rebecca's house?

Looking at the drawing, we can estimate the time to be between 4 and 5 hours. To determine exactly how long it takes, students could solve an equation or graph on a graphing calculator by entering the function rules.

a) $45 - 9h = 5$; $h = 4.44$

b) $Y_1 = 45 - 9h$ and $Y_2 = 5$.

Find the intersection of the two functions to determine that it takes about 4.44 hours for Daryl to get to Rebecca's house.

5. Write a sentence and a function rule describing Rebecca's distance from mile marker 0 in relation to elapsed time.

Rebecca starts at mile marker 5 and travels 11 miles per hour away from mile marker 0. A function rule for this would be $M = 5 + 11h$, where M is the mile marker and h is the number of hours.

6. Write a sentence and a function rule describing Daryl's distance from mile marker 0 in relation to elapsed time.

Daryl starts at mile marker 45, then travels 9 miles per hour towards mile marker 0. A function rule for this would be $M = 45 - 9h$, where M is the mile marker and h is the number of hours.

7. On graph below, plot the points from both tables and label each point with an ordered pair.

8. When are Rebecca and Daryl at the same place on the course? How can you tell from the graph?

They are at the same place, mile 27, after two hours. This is indicated on the graph by the intersection of the two lines.

9. Write an equation to determine how long it takes for Rebecca and Daryl to be at the same place on the course. What does this equation represent on the graph

Solve for h in the equation $5 + 11h = 45 - 9h$ to find the number of hours it takes for Rebecca and Daryl to be at the same place on the course. This equation represents the intersection point of the two functions that describe Rebecca's and Daryl's positions on the course over time.

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