Chapter 4: Quadratic Functions

A biologist was interested in the number of insect larvae present in water samples of various temperatures. He collected the following data:

Temperature (C°)	0	10	20	30	40	50
Population	20	620	920	920	620	20

1. Make a scatterplot of the data. Given the function $y = ax^2 + bx + c$, where b is 75, experiment with values of a and c to fit a quadratic function to your plot.

2. Write a verbal description of the relationship between the larvae population and the temperature of the water samples. What do the x- and y-intercepts mean? At what water temperature is the larvae population greatest?

3. The water sample is considered to be mildly contaminated but does not need to be treated if the larvae population is 300 or less. At what temperatures is the larvae population 300 or less? Explain.

4. Suppose that testing shows virtually no larvae present at 0°C, and the model for this situation is the function $y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(B)   investigate, describe, and predict the effects of changes in *a* on the graph of $y = ax^2 + c$;

(C)   investigate, describe, and predict the effects of changes in *c* on the graph of $y = x^2 + c$; and

(D)   analyze graphs of quadratic functions and draw conclusions.

Scaffolding Questions:

- How do the data in the table help you determine a reasonable window for your plot?

- What does the table indicate is a reasonable value for c? What does the shape of the graph tell you about the value of a?

- What is the function that most closely models this scatterplot?

- Experiment with values of a between –1 and –2.

- How can you use the graph of y = 300 to help you answer question 3?

Sample Solutions:

1.   Make a scatterplot of the data. Given the function $y = ax^2 + bx + c$, where *b* is 75, experiment with values of *a* and *c* to fit a quadratic function to your plot.

Scatterplot of larvae population vs. water temperature:
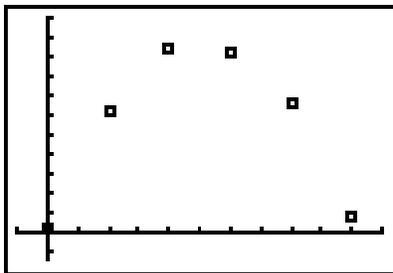
x-axis = Temperature

y-axis = Population

In looking for values of *c*, students may reason that, since $y = 20$ when $x = 0$, the value for *c* in $y = ax^2 + 75x + c$ is 20. Since the scatterplot shows the larvae population increasing and decreasing, a must be negative.
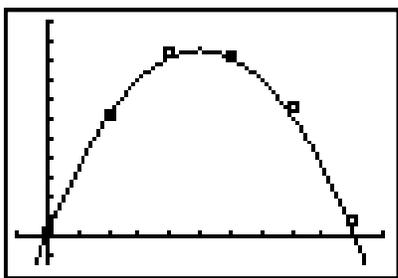
Looking for values of *a*, graphing $y = -1x^2 + 75x + 20$ gives a parabola that opens wider than the (original) plot appears. The following table shows values for this function. According to the values below, the larvae population is growing too fast compared to the original table of values.

Temperature (C°)	0	10	20	30	40	50
Population	20	670	1120	1370	1420	1270

Graphing $y = -2x^2 + 75x + 20$ gives a parabola that opens narrower than the (original) plot appears. The following table shows values for this function. According to the values below, the larvae population is growing too slowly compared to the original table of values.

Temperature (C°)	0	10	20	30	40	50
Population	20	570	720	470	−180	−1230

Trying a value close to $a = -1.5$, we find that a good fitting quadratic is $y = -1.5x^2 + 75x + 20$. The values in the table below (which are the values for this function) are the same values as those in the original table.

Temperature (C°)	0	10	20	30	40	50
Population	20	620	920	920	620	20

Using $a = -1.5$, then, is the most appropriate quadratic.

x-axis = Temperature

y-axis = Population

2. Write a verbal description of the relationship between the larvae population and the temperature of the water samples. What do the x- and y-intercepts mean? At what water temperature is the larvae population greatest?

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations the zeros of their related functions, and the horizontal intercepts (*x*–intercepts) of the graph of the function.

The *y*-intercept of the graph shows that at 0°C, there are 20 insect larvae in the water sample. As the temperature increases to 25°C, the population increases to about 957. This is shown by finding the coordinates of the vertex, (25, 957.5), using the graph. Then the population decreases to 0 at about 50°C (the *x*-intercept to the right of the origin).
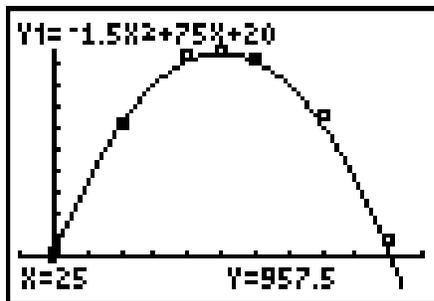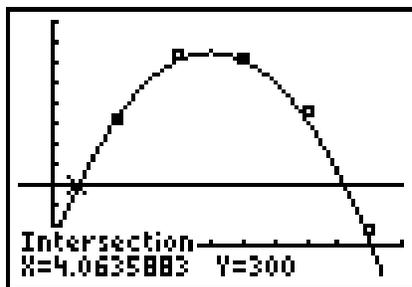

x-axis = Temperature

y-axis = Population

3. The water sample is considered to be mildly contaminated but does not need to be treated if the larvae population is 300 or less. At what temperatures is the larvae population 300 or less? Explain.

    By graphing *y* = 300 along with the population graph and finding the points of intersection, we can determine the temperatures when the population is no more than 300.

x-axis = Temperature

y-axis = Population

The graph shows that for temperatures up to 4°C, the

population of insect larvae is no more than 300. Since this point is 21 units to the left of the axis of symmetry, $x = 25$, the other intersection point is 21 units to the right of the axis of symmetry, and, therefore, is (46, 300). When the temperature is between 46°C and 50°C, the insect larvae population will again be no more than 300.

4. Suppose that testing shows virtually no larvae present at 0°C, and the model for this situation is the function $y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

   Since the first model ($y = -1.5x^2 + 75x + 20$) is in polynomial form and the second model (given in the question) is in factored form, rewrite the second model in polynomial form:

   $$y = -1.5x(x - 50)$$

   $$= -1.5x^2 + 75x$$

   Finding the x-intercepts and vertex of the graph either with a calculator or analytically, we find that the second model has x-intercepts (0, 0) and (50, 0) and vertex (25, 937.5). It is a translation of the first model, $y = -1.5x^2 + 75x + 20$, shifted down 20 units. The first model accounted for the 20 larvae observed to be present at 0°C. Compared to the first model, the second model underestimates the number of insect larvae present at any temperature by 20 larvae.

Extension Questions:

- What trends in the table and the graph tell you what is happening with the insect larvae in the water samples?

  The table shows that as the temperature increases to 20°C, the larvae population increases to 920, and then the population decreases to 20 at 50°C. The graph provides a more accurate picture. Since it is a parabola opening downward, the maximum number of larvae occurs at the vertex, which is when the temperature of the water is 25°C. The graph shows the population decreasing to 20 at 50°C and no larvae present at a fraction of a degree hotter than 50°C.

- How can you graphically investigate when the insect larvae population is no more than 300?

  *"No more than 300" means "less than or equal to 300." By graphing the line $y = 300$, we can find the portion of the larvae population graph that lies below the line graph, including the intersection points. We can do this using calculator features such as **trace** or **intersect**.*

- Suppose $y = -1.5x(x - 50) + 20$ is considered a usable model for predicting the number of insect larvae present in water samples, and a second round of experiments shows that the population at each of the previous temperatures in the table doubles. How will this affect the scatterplot and the function that models this new scatterplot?

  The new table will be

Temperature (C°)	0	10	20	30	40	50
Population	40	1240	1840	1840	1240	40

  The original scatterplot will be stretched vertically by a factor of 2 since the larvae population doubles. All of the y-values from the original function are multiplied by 2.

  The new function will be $y = 2 [-1.5x(x - 50) + 20]$ or $y = -3x(x - 50) + 40$.

- How does the new function in polynomial form compare with the original one?

  The coefficients and the constant in the new function are twice those of the original function.

  The original is $y = -1.5x(x - 50) + 20 = -1.5x^2 + 75x + 20$.

  The new function is $y = -3x(x - 50) + 40 = -3x^2 + 150x + 40$.