

A Safe Load

A-E Strand(s): Number: Measurement and Estimation. **Sample Courses:** Middle School Course 1, Middle School One-Year Advanced Course, Integrated 2, and Algebra I.

Topic/Expectation

N.B.1 Estimation and approximation

- c. Explain the phenomenon of rounding error, identify examples and, where possible, compensate for inaccuracies it introduces.
- d. Determine a reasonable degree of precision in a given situation.

G.A.3 Measurement

- d. Record measurements to reasonable degrees of precision, using fractions and decimals as appropriate.

Rationale

This task allows students to investigate the importance of using a reasonable degree of precision in a real life situation by allowing them to see how the degree of precision can dramatically affect the results.

Instructional Task

Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons. When the student performed the calculation, he used 1.4 as the approximation of $\sqrt{2}$. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student was shocked, as he had estimated that the bridge would hold ten times the weight that was applied to it when it collapsed. Complete the table below to investigate the effect of different possible approximations for $\sqrt{2}$. A column is included in the table for the square of the approximation used for $\sqrt{2}$ to help you notice how close to 2 the approximated square root is when squared.

Use the table to record your answers to questions 1–3.

1. Using the safe load formula above, calculate the weight that the engineering student thought the bridge could bear using 1.4 as his approximation of $\sqrt{2}$.
2. Calculate other weight values using an estimate of $\sqrt{2}$ with more decimal places.
3. Using a graphing calculator, calculate the weight value if $\sqrt{2}$ is entered into the calculator directly in the calculation instead of a rounded value.

Approximation of $\sqrt{2}$	Square of approximation	Process (formula for safe load) $1000(99 - 70\sqrt{2})$	Safe load value (tons)
1.4 (Student's Approximation)			
1.41			
1.414			
1.4142			
1.41421			
1.414213			
$\sqrt{2}$			

4. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.
5. Explain the engineering student's mistake. Write an explanation of why the bridge collapsed.

Discussion/Further Questions/Extension

Background: Students may need to review what 'precision' means in a mathematical context—that is, precision refers to the number of significant digits, whereas accuracy refers to how close a calculated value is to the actual value. A value, then, can be precise without being accurate or accurate without being precise. Ideally, you want a value to be both precise and accurate. Note that in the table, the values for $\sqrt{2}$ are increasingly precise and also increasingly accurate.

Calculator precision: Most good calculators approximate irrational numbers to 14 decimal places of precision internally and then report about 12 places on the screen. Sometimes you can use a trick to get one or two more decimal places of the approximation to show by subtracting the whole number part. For $\sqrt{2}$ on a calculator, compare the first value returned with $(\sqrt{2}) - 1$. Make sure your calculator is in float mode for the number of decimal places displayed.

Computer precision: Software spreadsheet programs can often provide more precision than calculators, since they are not limited by screen size. For example, one version of Excel allows 30 decimal places to show in a spreadsheet cell. Using such a program may be advantageous when working with more than 12 digits of precision, especially when the context requires as exact an answer as possible.

Sample Solution

- Using the safe load formula above, calculate the weight that the engineering student thought the bridge could bear using 1.4 as his approximation of $\sqrt{2}$.

See table below.

- Calculate other weight values using an estimate of $\sqrt{2}$ with more decimal places.

See table below.

- Using a graphing calculator, calculate the weight value if $\sqrt{2}$ is entered into the calculator directly in the calculation instead of a rounded value.

See table below.

Note the safe load value using 1.4 as an approximation for $\sqrt{2}$ is 1000 tons, while the value using $\sqrt{2}$ on a graphing calculator is 5.0506 tons. (Most good calculators approximate irrational numbers to 14 decimal places.)

Approximation for $\sqrt{2}$	Square of approximation $(\sqrt{2})^2$	Process (formula for safe load) $1000(99 - 70\sqrt{2})$	Safe load value (tons)
1.4	1.96	$1000(99 - 70 \cdot 1.4)$	1000
1.41	1.9881	$1000(99 - 70 \cdot 1.41)$	300
1.414	1.999396	$1000(99 - 70 \cdot 1.414)$	20
1.4142	1.99996164	$1000(99 - 70 \cdot 1.4142)$	6
1.41421	1.9999899241	$1000(99 - 70 \cdot 1.41421)$	5.3
1.414213	1.999998409369	$1000(99 - 70 \cdot 1.414213)$	5.09
$\sqrt{2}$	2	$1000(99 - 70\sqrt{2})$	5.0506

- What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

Answers may vary. Using an approximation to the millionths place (6 decimal places) would accurately show that the bridge can hold between 5 and 5.1 tons. However, using a 14-decimal-place approximation for $\sqrt{2}$ from a graphing calculator gives an even better estimate of 5.0506 tons.

- Explain the engineering student's mistake. Write an explanation of why the bridge collapsed.

Answers will vary. A possible answer is that the engineering student failed to realize the eventual impact of using a one decimal approximation for $\sqrt{2}$. When using 1.4 as an approximation for $\sqrt{2}$, the formula yields: $1000(99 - 70(1.4)) = 1000(99 - 98) = 1000(1) = 1000$. This would mean that the bridge is calculated to hold a load of 1000 tons. As the number of decimal places in the approximation of $\sqrt{2}$ is increased, a larger value is subtracted from 99. The result of this subtraction is a smaller value that has greater precision (i.e., more decimal places). The seemingly insignificant variation in the result from this subtraction becomes significant when the difference is multiplied by 1000, yielding a large change in the final product.

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